PERGAMON

International Journal of Multiphase Flow 25 (1999) 739–751

International Journal of Multiphase Flow

The inertial lift on an oscillating sphere in a linear shear flow

Evgeny S. Asmolov[a], J.B. McLaughlin[b,*]

[a]Zhukovsky Central Aero-Hydrodynamics Institute, Zhukovsky, Moscow Region 140160, Russia
[b]Department of Chemical Engineering, Clarkson University, Potsdam, NY 13699-5705, USA

Received 11 December 1997; received in revised form 18 October 1998

Abstract

In a shear flow, a small sphere may experience a lift force due to fluid inertia. Most previous workers assumed that the particle was stationary so that they could treat the fluid motion as steady. In spite of this, the results of previous analyses have generally been applied to problems in which particles move in an unsteady fashion. This paper presents the results of singular perturbation calculations of the lift on a sphere in a linear shear flow. The velocity of the sphere oscillates sinusoidally in time. Although the problem is idealized, the results provide some physical understanding of the effects of unsteadiness and the frequency regime in which one may assume quasisteady conditions. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Lift; Oscillation; Sphere

1. Introduction

Small particles can experience lift forces as a result of fluid inertia. Lift forces can cause particles to cross streamlines in laminar flows. There are a number of engineering applications where lift forces are known or suspected to play an important role. Williams et al. (1996) showed that the lift force plays a role in field flow fractionation. Asmolov (1995) showed that the lift force on the particles in a dusty gas flow plays an important role in determining the point of flow separation over blunt bodies. Asmolov and Manuilovich (1998) argued that the lift force may play a role in the transition to turbulence for dusty gas boundary layers. Zeng et al. (1993) discussed the role of lift forces in bubble detachment from solid surfaces in flow

* Corresponding author. Tel.: +1-315-268-6663; fax: +1-315-268-6654; e-mail: jmclau@clarkson.edu.

PII: S0301-9322(98)00063-9

boiling. Lift forces also play a role in the deposition on and accumulation of particles near solid surfaces in turbulent flows (Kallio and Reeks, 1989; McLaughlin, 1989; Chen and McLaughlin, 1996).

Most of the existing theoretical work on lift forces for small particles makes use of perturbation expansions based on the particle Reynolds numbers. Saffman (1965) used singular perturbation theory to derive an expression for the lift force acting on a small sphere in an unbounded linear shear flow. He treated the Reynolds numbers of the particle based on the shear rate and the slip velocity as small parameters.

Cox and Brenner (1968) considered the lift on small particles in wall-bounded flows. They showed that, provided that the particle was close enough to the wall, one could use regular perturbation theory to compute the leading order expression for the lift force. Cox and Hsu (1977) derived results for the lift force on a small particle in a wall-bounded parabolic shear flow. Their analysis is valid if the wall lies within the "inner" region of the disturbance flow created by the particle. In the inner region, inertial effects are small compared with viscous effects.

Asmolov (1998) used singular perturbation methods to extend previous results on parabolic channel flows to higher Reynolds numbers. His paper contains references to other work on lift forces on wall-bounded flows.

Saffman's formulation of the problem for an unbounded linear shear flow is general. However, to obtain quantitative results, he assumed that the particle Reynolds number based on the shear rate, Re_G, and the particle Reynolds number based on the slip velocity, Re_s, satisfied the inequality $Re_G^{1/2} >> Re_s$. Asmolov (1990) and McLaughlin (1991, 1993) removed this restriction and included the effect of a distant wall on the lift force. Their treatment of the wall does not require that the wall must lie within the inner region of the particle disturbance flow. Thus, their results generalize the results of Cox and Hsu.

In many applications of interest, the motion of the particle and/or the fluid is unsteady. As an example, in their stability analysis, Asmolov and Manuilovich (1998) assumed that the wavelength of the Tollmien–Schlichting wave was of the order of the local thickness of the boundary layer and small compared with the characteristic distance from the leading edge of a plate. This means that the shear rate varies in time more slowly than the disturbance slip velocity. They argued that, in the critical layer of the TS wave, it may be important to consider the effect of unsteadiness on the lift force. Since the time dependence of the TS wave is approximately sinusoidal, the results of the present paper are relevant to the analysis presented by Asmolov and Manuilovich.

In spite of the fact that the various lift force formulas are often used in situations where the shear flow and/or particle motion are unsteady, virtually all previous work on lift forces has treated both the flow and the motion of the particle as steady. An exception is the work of Miyazaki et al. (1995). They developed an induced force field formulation that can be used to treat unsteadiness. They presented results for the stationary case and the high frequency limit and showed, in the former case, that their result was consistent with Saffman's result.

In the present paper, results will be derived for the lift on a spherical particle that oscillates along the direction of simple shear flow. Conventional singular perturbation methods will be used to obtain the results. An analytical result that is consistent with Miyazaki et al.'s result will be derived in the high frequency limit. The role and significance of various length scales will be discussed and the regime in which the quasisteady assumption is accurate will be identified.

Lovalenti and Brady (1993) considered the force on a sphere in a uniform flow with small amplitude oscillations at finite Reynolds numbers. However, in this case, the lift force on the particle vanishes. Their work complements the present paper in that they provided results for the time-dependent drag on the sphere.

2. Review of Saffman's problem

In this section, Saffman's analysis of the lift on a small particle in a steady shear flow will be briefly discussed. Saffman derived the lowest order expression for the lift acting on a rigid sphere in a steady, unbounded linear shear flow. Saffman assumed that the sphere moved parallel to the streamlines of the undisturbed flow, but with a different velocity.

Several Reynolds numbers are needed to characterize the disturbance flow created by the sphere. One Reynolds number, Re_s, is based on the sphere diameter and the difference between the velocity of the center of the sphere and the undisturbed velocity of the fluid at the same point:

$$Re_s = \frac{|\upsilon_s|d}{\nu} \tag{1}$$

where d is the diameter of the sphere and ν is the kinematic viscosity of the fluid. A second Reynolds number, Re_G, is based on the sphere diameter and the local shear rate of the undisturbed flow:

$$Re_G = \frac{|G|d^2}{\nu} \tag{2}$$

where G is the local shear rate of the mean flow. (It is assumed that the mean flow is unidirectional for simplicity.) In general, one would need an additional Reynolds number based on the angular velocity of the sphere. However, if the sphere is torque-free, one can relate the angular velocity to the local shear rate.

Saffman assumed that Re_G and Re_s are both small and that the ratio $Re_G^{1/2}/Re_s$ is large compared with unity. He obtained the following expression for the lift force:

$$F_l = 6.46\mu a^2 \upsilon_s \sqrt{\frac{|G|}{\nu}}\,\mathrm{sign}(G) \tag{3}$$

In Eq. (3), μ is the dynamic viscosity of the fluid, a is the radius of the sphere, and $-\upsilon_s \mathbf{e}_3$ is the velocity of the sphere's center relative to the undisturbed fluid. The undisturbed fluid velocity is $Gx\mathbf{e}_3$. The quantity υ_s will be referred to as the "slip velocity" in the rest of this paper. If $Gv_s > 0$, the lift force points in the positive x-direction. If the sphere is not constrained by another force, it will migrate in the positive x-direction. Saffman assumed that the migration velocity is small enough that it may be neglected.

The Saffman lift force is caused by a transverse flow that originates at large distances from the sphere in an Oseen-like region. In this region, the magnitude of the convective term in the Navier–Stokes equation is comparable with the magnitude of the viscous term. The length that characterizes the Oseen-like region is the Saffman length,

$$L_{\rm G} = \left(\frac{\nu}{|G|}\right)^{1/2} \tag{4}$$

For Saffman's analysis to be valid, the Saffman length must be large compared with the particle diameter but small compared to the Stokes length, $L_{\rm s}$, defined by

$$L_{\rm s} = \frac{\nu}{|\upsilon_{\rm s}|} \tag{5}$$

With the above assumptions, a singular perturbation expansion shows that the lift force may be obtained to leading order by approximating the sphere by a point force. The singular perturbation expansion considers an inner region in which the radial distance from the center of the sphere, r, is $O(a)$, and an outer region in which r is $O(L_{\rm G})$. The velocity of the fluid, $\mathbf{v}'$, may be written as

$$\mathbf{v}' = \mathbf{U} + \mathbf{v} \tag{6}$$

where $\mathbf{U}$ is the undisturbed fluid velocity and $\mathbf{v}$ is the disturbance created by the sphere. For the problem of interest, in a frame of reference in which the sphere is at rest at $x = 0$,

$$\mathbf{U} = (Gx + \upsilon_{\rm s})\mathbf{e}_3 \tag{7}$$

In the outer region, one linearizes the Navier–Stokes equation in the disturbance flow and neglects the contribution to the convective term involving $\upsilon_{\rm s}$. The latter approximation is valid provided that $L_{\rm G}\ /L_{\rm s} < < 1$. One then solves the following linearized form of the Navier–Stokes equation for the disturbance velocity, $\mathbf{v}$:

$$Gx\frac{\partial \mathbf{v}}{\partial z} + G\upsilon_1\mathbf{e}_3 = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v} - \frac{F}{\rho}\delta(\mathbf{r})\mathbf{e}_3 \tag{8}$$

Asmolov (1990) and McLaughlin (1991) removed the restriction on the relative sizes of the two Reynolds numbers by including an Oseen approximation to the convective term involving the slip velocity in the linearized Navier–Stokes equation. The lift force may be written in the following form:

$$F_{\rm l} = \frac{9}{\pi}\mu a^2 \upsilon_{\rm s}\sqrt{\frac{|G|}{\nu}}J(\epsilon)\,{\rm sign}(G) \tag{9}$$

where J is a dimensionless function of the dimensionless parameter ϵ defined by

$$\epsilon = \frac{Re_{\rm G}^{1/2}}{Re_{\rm s}}\,{\rm sign}(G\upsilon_{\rm s}) \tag{10}$$

McLaughlin calculated J and found that it goes to zero as ϵ goes to zero. The Saffman formula over-estimates the magnitude of J for all finite values of ϵ. In the limit $\epsilon >> 1$, J monotonically increases to the value 2.254 (to four digits) and Eq. (9) reduces to Eq. (3).

3. Formulation of the problem

Let us consider a rigid sphere that executes simple harmonic motion in the z-direction in a steady linear shear flow $\mathbf{u} = Gx\mathbf{e}_3$. It is convenient to pose the problem in a frame of reference in which the sphere is stationary. The equation for the disturbance flow created by the particle, $\mathbf{v}$, is

$$\frac{\partial \mathbf{v}}{\partial t} + Gx\frac{\partial \mathbf{v}}{\partial z} + Gv_1\mathbf{e}_3 = -\frac{1}{\rho}\nabla \mathbf{p} + \nu\nabla^2\mathbf{v} - \frac{F}{\rho}\delta(\mathbf{r})\mathbf{e}_3 \tag{11}$$

where

$$F = \tilde{F}e^{-i\omega t} \tag{12}$$

$$\mathbf{v} = \tilde{\mathbf{v}}e^{-i\omega t} \tag{13}$$

$$p = \tilde{p}e^{-i\omega t} \tag{14}$$

and it is understood that one must take the real part of $\mathbf{v}$, p and F to obtain the physical values. In deriving Eq. (11), the Saffman assumption, $|\epsilon| >> 1$, is made. In the more general case where $|\epsilon| = O(1)$, the uniform flow term, v_s, in Eq. (7) introduces a nonlinear coupling between different frequency components in the disturbance flow created by the particle. However, in the regime considered by Saffman, the different frequency components are uncoupled.

One may rewrite Eq. (11) in the following form:

$$-i\omega\tilde{\mathbf{v}} + Gx\frac{\partial \tilde{\mathbf{v}}}{\partial z} + G\tilde{v}_1\mathbf{e}_3 = -\frac{1}{\rho}\nabla\tilde{p} + \nu\nabla^2\tilde{\mathbf{v}} - \frac{\tilde{F}}{\rho}\delta(\mathbf{r})\mathbf{e}_3 \tag{15}$$

The flow is assumed to be incompressible,

$$\nabla\cdot\mathbf{v} = \nabla\cdot\tilde{\mathbf{v}} = 0 \tag{16}$$

For an unbounded flow, one can use the Fourier transform technique devised by Saffman (1965) to obtain the lift force from Eq. (11). Saffman pointed out that it is not necessary to solve Eq. (8) to obtain the lift force. One need only calculate the x-component of the velocity at the origin. In essence, the transverse flow looks like a uniform flow in the x-direction to the particle. To obtain the lift force from the velocity, one uses the Stokes drag law.

The time-dependence of the flow introduces a third length, L_ω:

$$L_\omega = \left(\frac{\nu}{\omega}\right)^{1/2} \quad (17)$$

The character of the disturbance flow depends on the ratio L_ω/L_G in the limit $|\epsilon| >> 1$. The limit $L_\omega/L_G >> 1$ may be thought of as the limit in which the oscillation period is very large compared with the viscous time scale. If $L_\omega/L_G >> 1$, the disturbance flow should be the same as in the steady problem considered by Saffman. However when the ratio is order unity, deviations from the steady-state result may be anticipated. Provided that $L_\omega >> a$, $\tilde{F}$ may be approximated by the steady Stokes drag law:

$$\tilde{F} = -6\pi\mu a\tilde{\upsilon}_s \quad (18)$$

where $\upsilon_s = \upsilon_s e^{-i\omega t}$.

If the condition $|\epsilon| >> 1$ is not satisfied, one must include a convective term involving υ_s in Eq. (15). This introduces an explicit time dependence in the coefficient of the disturbance velocity. For $|\epsilon| >> 1$, one can use Floquet theory to compute the leading corrections in the small parameter $1/|\epsilon|$. However, for values of $|\epsilon|$ that are order unity, one must use numerical methods to solve the problem. Therefore, this paper will present a result for the Fourier transform of the lift force only in the Saffman limit. In the Saffman regime, it is possible to express the Fourier transform in terms of the dimensionless parameter ω/G (or L_G/L_ω).

4. Solution for an unbounded fluid

In this section, the solution of Eq. (15) for the case of an unbounded fluid will be obtained. The mathematical manipulations used to obtain the solution follow Saffman (1965) and McLaughlin (1991). It is convenient to introduce the Fourier transforms of the velocity field and the pressure field:

$$\tilde{\mathbf{v}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\mathbf{u}} e^{i(k_1 x + k_2 y + k_3 z)} \mathrm{d}k, \quad (19)$$

and

$$\tilde{p} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\Pi} e^{i(k_1 x + k_2 y + k_3 z)} \mathrm{d}k. \quad (20)$$

By substituting the Fourier transforms in Eqs. (19) and (20) into Eq. (15), one can obtain an ordinary differential equation for $\tilde{\mathbf{u}}$ which takes the form

$$-i\omega\tilde{\mathbf{u}} = -ik\tilde{\Pi}/\rho - \nu k^2\tilde{\mathbf{u}} - G\tilde{u}_1\mathbf{e}_3 + Gk_3\frac{\partial\tilde{\mathbf{u}}}{\partial k_1} - \frac{\tilde{F}}{8\pi^3\rho}\mathbf{e}_3 \quad (21)$$

Using the incompressibility condition, $\mathbf{k}\cdot\tilde{\mathbf{u}} = 0$, the expression for $\tilde{u}_1$ can now be written in the form

$$\tilde{u}_1 = \frac{3}{4\pi^2}\frac{\nu a\tilde{v}_s k_3}{Gk^2}\int_0^\infty e^{(\psi'-\psi)}(\zeta k_3 + k_1)\,d\zeta \tag{22}$$

where

$$\psi' - \psi = -\frac{\nu}{3G}k_3^2\zeta^3 - \frac{\nu}{G}k_1k_3\zeta^2 - \frac{\nu}{G}k^2\zeta + \frac{i\omega}{G}\zeta \tag{23}$$

The expression for $\tilde{u}_1$ in Eq. (22) is valid regardless of the sign of Gk_3. The expression on the right hand side of Eq. (23) is identical to the expression used by Saffman except for the last term.

If the expression for $\tilde{u}_1$ in Eq. (22) is substituted into Eq. (19), the values of the disturbance flow velocity can be calculated at any point in space. As $r = (x^2 + y^2 + z^2)^{1/2}$ approaches zero, the disturbance flow must approach that of a Stokeslet solution, and, in order to determine the inertial migration velocity, it is necessary to compute the difference between the disturbance flow and a Stokeslet flow and take the limit in which r goes to zero. The manipulations needed to obtain the inertial migration velocity are the same as those described by Saffman (1965) and McLaughlin (1991). When the dimensionless wavevector, $\mathbf{q} = (\nu/G)^{1/2}\mathbf{k}$, is introduced, it can be shown that

$$\tilde{F}_1 = \frac{9}{2\pi}\rho a^2\tilde{v}_s(G\nu)^{1/2}I(\Omega) \tag{24}$$

where I is the four-dimensional integral that is defined below:

$$I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\zeta\left\{\frac{q_3^2}{q^2} - \frac{q_1q_3(2q_1q_3 + \zeta q_3^2)}{q^4}\right\} + i\frac{q_1q_3}{q_4}\Omega\right]e^{-(q_3^2\zeta^3/3 + q_1q_3\zeta^2 + q^2\zeta)}e^{i\Omega\zeta}\,d\zeta\,d\mathbf{q}, \tag{25}$$

and $\Omega = \omega/G$. Integration over $|\mathbf{q}|$ yields $I = 2J$, where

$$J = \frac{\pi^{1/2}}{4}\int_0^{2\pi}\int_0^1\int_0^\infty\Big[\zeta\{s^2 - 2s^2(1-s^2)\cos^2\phi - \zeta s^3(1-s^2)^{1/2}\cos\phi\}A^{-3} + 2i\Omega A^{-1}s(1-s^2)^{1/2}\cos\phi\Big]e^{i\Omega\zeta}\,d\zeta\,ds\,d\phi \tag{26}$$

In Eq. (26),

$$A^2 = \frac{s^2\zeta^3}{3} + s(1-s^2)^{1/2}\cos\phi\,\zeta^2 + \zeta \tag{27}$$

In the above equations, $s = \cos\theta$ and ϕ and θ denote the angular coordinates in a spherical coordinate system in Fourier space. The symbol J is introduced to facilitate comparisons with McLaughlin's (1991) work.

In general, the integrals in Eq. (26) must be evaluated numerically. First, the asymptotic limits in which $\Omega << 1$ and $\Omega >> 1$ will be explored. The regime $\Omega >> 1$ has an upper bound because of the restriction $\omega << \nu/a^2$.

When the integral over $|\mathbf{q}|$ is performed, the expression for I in Eq. (25) can be written as

$$I = \frac{\pi^{1/2}}{2}\int_0^{2\pi}\int_0^1 [I_1\{s^2 - 2s^2(1-s^2)\cos^2\phi\} - I_2 s^3(1-s^2)^{1/2}\cos\phi + 2iI_3 s(1-s^2)^{1/2} \times \cos\phi]\,\mathrm{d}s\,\mathrm{d}\phi \tag{28}$$

where

$$I_1(s,\phi) = \int_0^\infty \zeta A^{-3} \mathrm{e}^{i\Omega\zeta}\,\mathrm{d}\zeta \tag{29}$$

$$I_2(s,\phi) = \int_0^\infty \zeta^2 A^{-3} \mathrm{e}^{i\Omega\zeta}\,\mathrm{d}\zeta \tag{30}$$

$$I_3(s,\phi) = \int_0^\infty \Omega A^{-1} \mathrm{e}^{i\Omega\zeta}\,\mathrm{d}\zeta \tag{31}$$

When $\Omega << 1$, it is straightforward to expand the expression of I in powers of Ω. The coefficients of the expansion are dimensionless integrals that are evaluated numerically. The result, to first order, is

$$J = 2.254 + 3.894\Omega i \tag{32}$$

When $\Omega = 0$, the result in Eq. (32) reduces to Saffman's result.

To obtain the asymptotic behavior of I_1, I_2 and I_3 for $\Omega >> 1$, it is convenient to introduce a new variable: $\xi = \zeta\Omega$. Then,

$$A = \left[\frac{\xi}{\Omega}\left(1 + \frac{\xi s(1-s^2)^{1/2}\cos\phi}{\Omega} + \frac{\xi^2 s^2}{3\Omega^2}\right)\right]^{1/2} \tag{33}$$

and the leading order terms in the expansions of I_1 and I_3 are

$$I_1^0 = \Omega^{-1/2} P \tag{34}$$

and

$$I_3^0 = \Omega^{1/2} P \tag{35}$$

where

$$P = \int_0^\infty \xi^{-1/2} \mathrm{e}^{i\xi}\,\mathrm{d}\xi = \left(\frac{\pi}{2}\right)^{1/2}(1+i) \tag{36}$$

Table 1
Numerical results for J

Ω	J_r	J_i
0.000	2.254	0.000
0.001	2.254	0.003886
0.010	2.252	0.03857
0.100	2.159	0.3327
0.200	1.989	0.556
0.300	1.807	0.699
0.400	1.634	0.785
0.500	1.477	0.830
0.600	1.340	0.849
0.700	1.221	0.850
0.800	1.122	0.844
0.900	1.033	0.822
1.000	0.960	0.801
1.100	0.896	0.777
1.200	0.842	0.753
1.300	0.796	0.729
1.400	0.756	0.706
1.500	0.721	0.684
1.600	0.690	0.663
1.700	0.663	0.643
1.800	0.640	0.624
1.900	0.618	0.607
2.000	0.599	0.591
2.500	0.526	0.525
3.000	0.477	0.476
3.500	0.439	0.440
4.000	0.410	0.410
5.000	0.366	0.366
7.000	0.309	0.310
8.000	0.288	0.288
9.000	0.271	0.271
10.000	0.258	0.258

while $I_2 \sim \Omega^{-3/2}$. The integral of the term proportional to I_3^0 with respect to ϕ yields 0. Thus, one has to evaluate the first order term:

$$I_3^1 = \Omega^{1/2} \int_0^\infty \left[\left(1 + \frac{\xi s (1-s^2)^{1/2} \cos\phi}{\Omega} \right)^{-1/2} - 1 \right] \xi^{-1/2} \mathrm{e}^{i\xi} \, \mathrm{d}\xi \tag{37}$$

If one integrates by parts and makes an expansion in powers of Ω^{-1}, one obtains

$$I_3^1 = \frac{1}{4i} \Omega^{-1/2} P s (1-s^2)^{1/2} \cos\phi \tag{38}$$

Substituting the expressions for I_1^0 and I_3^1 into the equation for I and integrating with respect to ϕ and s yields

$$I = \frac{7\pi^2}{30(2\Omega)^{1/2}}(1+i) \tag{39}$$

or

$$J = \frac{7\pi^2}{60(2\Omega)^{1/2}}(1+i) \tag{40}$$

which is consistent with the numerical results to be presented. The results in Eqs. (39) and (40) are consistent with the high frequency results presented by Miyazaki et al. (1995).

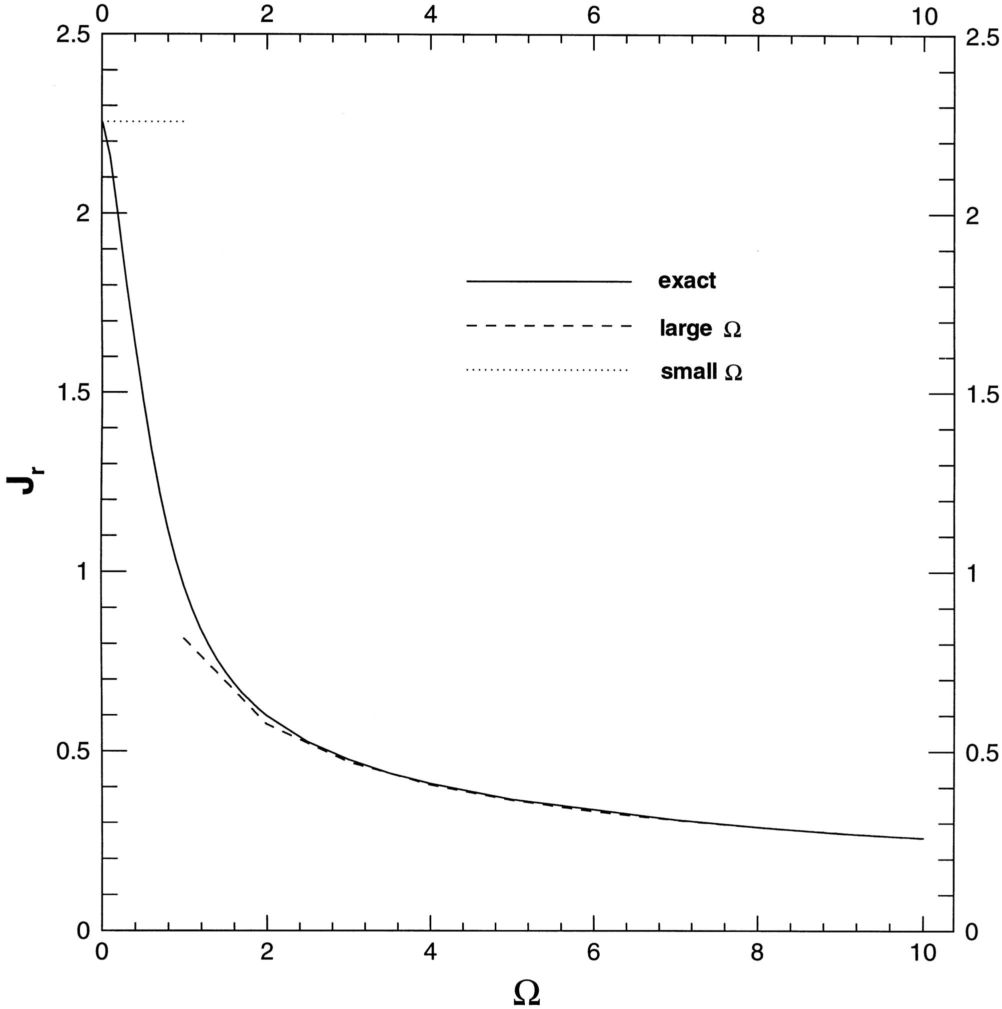

Fig. 1. Real part of J compared with asymptotic approximations.

When Ω is $O(1)$, I must be evaluated numerically. The expression for I in Eq. (28) was evaluated by numerical integration. The results for $J = I/2$ are given in Table 1.

Figures 1 and 2 show the "exact" numerical results for the real and imaginary parts of J as well as the asymptotic forms for small and large Ω.

In applications, it is often convenient to have an analytical fit for the lift force. Both the real and imaginary parts of J may be approximated by a fit of the following form:

$$J_{r,i} = \frac{a_0 + a_1\Omega + a_2\Omega^2 + a_3\Omega^3 + a_4\Omega^{7/2}}{1 + a_5\Omega + a_6\Omega^2 + a_7\Omega^3 + a_8\Omega^4} \tag{41}$$

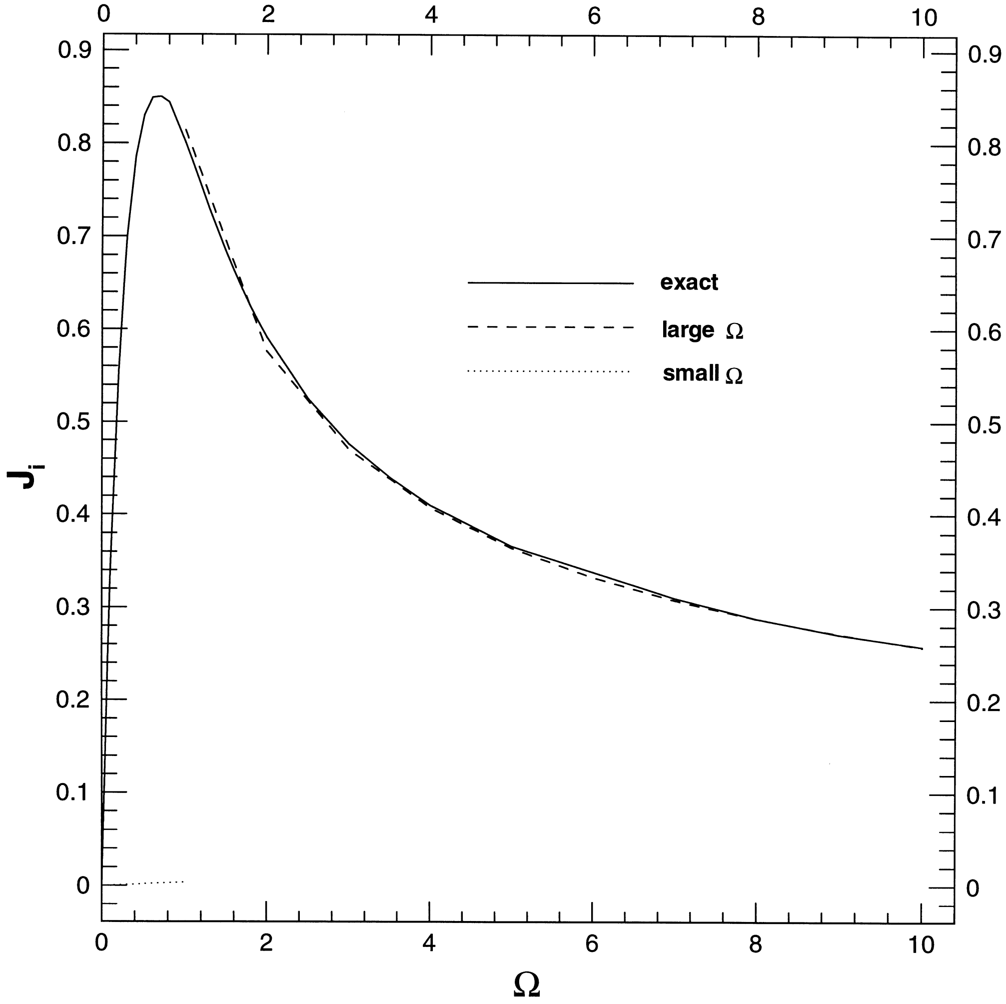

Fig. 2. Imaginary part of J compared with asymptotic approximations.

Table 2
Constants in fits for J_r,i

a_i	J_r	J_i
a_0	2.254	0
a_1	4.528	3.378
a_2	−2.378	1.391
a_3	−0.648	−0.575
a_4	2.079	1.139
a_5	2.009	0.523
a_6	4.048	5.199
a_7	−3.545	−1.396
a_8	2.554	1.399

The constants in Eq. (41) are given in Table 2 for the real and imaginary parts of J. The largest errors of the fit equations are 0.008 in magnitude for the real part and 0.0014 for the imaginary part.

5. Conclusion

The main results of this paper are the values of $J = I/2$ given in Table 1. These values and Eq. (24) determine the lift force on a sphere. The results are valid in the strong shear regime considered by Saffman ($Re_G^{1/2} >> Re_s$). It is also assumed that the characteristic length L_ω is large compared with the diameter of the sphere. The effect of unsteadiness can be characterized in terms of L_ω and the Saffman length, L_G. For $L_\omega << L_G$, the lift force is well approximated by the Saffman formula. However, when L_ω is comparable to or smaller than L_G, the lift force is smaller in magnitude and out of phase with the force that one would predict from Saffman's formula.

The lift force arises from fluid motion at large distances from the sphere. The Oseen-like form in Eq. (11) is linear in the disturbance flow. Thus, the monochromatic results reported in this paper could be superposed to obtain the lift force for spheres executing more complicated unidirectional motions.

Acknowledgements

One of the authors (E.S.A) would like to acknowledge the Russian Foundation for Fundamental Research (grant no. 96-01-01245) for financial support. Some of the techniques and motivation for this work were developed with support (J.B.M.) from Dr Robert Price's program at the US Department of Energy under grant DE-FG02-88ER13919.

References

Asmolov, E.S., 1990. Dynamics of a spherical particle in a laminar boundary layer. Fluid Dynamics 25, 886–890.

Asmolov, E.S., 1995. Dusty-gas flow in a laminar boundary layer over a blunt body. Journal of Fluid Mechanics 305, 29–46.

Asmolov, E.S., 1998. The inertial lift on a spherical particle in a plane Poiseuille flow at large channel Reynolds number. Journal of Fluid Mechanics (in press).

Asmolov, E.S., Manuilovich, S.V., 1988. Stability of dusty-gas laminar boundary layer on a flat plate. Journal of Fluid Mechanics (in press).

Chen, M., McLaughlin, J.B., 1996. A new correlation for the aerosol deposition rate in vertical ducts. Journal of Colloid Interface Science 169, 437–455.

Cox, R.G., Brenner, H., 1968. The lateral migration of solid particles in Poiseuille flow: I. Theory. Chemical Engineering Science 23, 147–173.

Cox, R.G., Hsu, S.K., 1977. The lateral migration of solid particles in a laminar flow near a plane. International Journal of Multiphase Flow 3, 201–222.

Kallio, G.A., Reeks, M.W., 1989. A numerical simulation of particle deposition in turbulent boundary layers. International Journal of Multiphase Flow 15, 433–446.

Lovalenti, P.M., Brady, J.F., 1993. The force on a sphere in a uniform flow with small-amplitude oscillations at finite Reynolds number. Journal of Fluid Mechanics 256, 607–614.

McLaughlin, J.B., 1989. Aerosol particle deposition in numerically simulated channel flow. Physics of Fluids A 1, 1211–1224.

McLaughlin, J.B., 1991. Inertial migration of a small sphere in linear shear flows. Journal of Fluid Mechanics 224, 261–274.

McLaughlin, J.B., 1993. The lift on a small sphere in wall-bounded linear shear flows. Journal of Fluid Mechanics 246, 249–265.

Miyazaki, K., Bedeaux, D., Bonet, Avalos J., 1995. Drag on a sphere in slow shear flow. Journal of Fluid Mechanics 296, 373–390.

Saffman P.G., 1965. The lift on a small sphere in a slow shear flow, Journal of Fluid Mechanics, 22, 385–400, Corrigendum, 1968, 31, 624.

Williams, P.S., Moon, M.H., Xu, Y., Giddings, J.C., 1996. Effect of viscosity on retention time and hydrodynamic lift forces in sedimentation/steric field-flow fractionation. Chemical Engineering Science 51, 4477–4488.

Zeng, L.Z., Klausner, J.F., Bernhard, D.M., Mei, R., 1993. A unified model for the prediction of bubble detachment diameters in boiling systems—II. Flow boiling. International Journal of Heat Mass Transfer 36, 2271–2279.